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The tachyon at the end of the universe

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Abstract: We show that a tachyon condensate phase replaces the spacelike singularity in certain cosmological and black hole spacetimes in string theory. We analyze explicitly a set of examples with flat spatial slices in various dimensions which have a winding tachyon condensate, using worldsheet path integral methods from Liouville theory. In a vacuum with no excitations above the tachyon background in the would-be singular region, we analyze the production of closed strings in the resulting state in the bulk of spacetime. We find a thermal result reminiscent of the Hartle-Hawking state, with tunably small energy density. The amplitudes exhibit a self-consistent truncation of support to the weakly-coupled small-tachyon region of spacetime. We argue that the background is accordingly robust against back reaction, and that the resulting string theory amplitudes are perturbatively finite, indicating a resolution of the singularity and a mechanism to start or end time in string theory. Finally, we discuss the generalization of these methods to examples with positively curved spatial slices.

Keywords: Superstrings and Heterotic Strings, Tachyon Condensation, Black Holes in String Theory, Cosmology of Theories beyond the SM.

Contents

1. Introduction

Closed string tachyon condensation affects the dynamics of spacetime in interesting and tractable ways in many systems [1−5]. In this paper, we study circumstances in which closed string tachyon condensation plays a crucial role in the dynamics of a system which has a spacelike singularity at the level of general relativity. The singular region of the spacetime is replaced by a phase of tachyon condensate which lifts the closed string degrees of freedom, effectively ending ordinary spacetime.

We will focus primarily on a simple set of examples with shrinking circles, in which we can make explicit calculations exhibiting this effect. Before specializing to this, let us start by explaining the relevant structure of the stringy corrections to spacelike singularities appearing in a more general context. Much of this general discussion appeared earlier in [6].[1] Consider a general relativistic solution approaching a curvature singularity in the

[1] The possibility of applying the worldsheet mass gap in higher dimensional generalizations of [3] was also independently suggested by A. Adams and M. Headrick.

past or future. The metric is of the form

$$ds^2 = G_{\mu\nu} dx^\mu dx^\nu = -(dx^0)^2 + R_i(x^0)^2 d\Omega_i^2 + ds_\perp^2 \tag{1.1}$$

with $R_i(x^0) \to 0$ for some i at some finite time. Here Ω_i describe spatial coordinates whose scale factor is varying in time and $ds_\perp^2$ describes some transverse directions not directly participating in the time dependent physics.

In the large radius regime where general relativity applies, the background (1.1) is described by a worldsheet sigma model with action in conformal gauge

$$S_0 \equiv \frac{1}{4\pi\alpha'} \int d^2\sigma \; G_{\mu\nu}(X) \partial_a X^\mu \partial^a X^\nu + \text{fermions} + \text{ghosts} \,. \tag{1.2}$$

Here we are considering a type II or heterotic string with worldsheet supersymmetry in order to avoid bulk tachyons.

As the space shrinks in the past or future, at leading order in α' (i.e. in GR) the corresponding sigma model kinetic terms for Ω develop small coefficients, leading to strong coupling on the worldsheet. This raises the possibility of divergent amplitudes in the first quantized worldsheet path integral description from lack of suppression from the action. This would correspond also to the development of an effectively strong coupling in the spacetime theory as the size of the Ω directions shrink.

However, there is more to the story in string theory. Let us first consider the sigma model on the angular geometry at fixed time X^0. When any of the radii R_i in (1.1) is of order string scale, this strongly coupled sigma model is very different from the free flat-space theory. In particular, it can dynamically generate a mass gap in the IR [7, 8]. In such cases the quantum effective action in this matter sigma model has terms of the form $\int d^2\sigma \mathcal{O}_\Delta \Lambda^{2-\Delta}$ where $\Delta < 2$ is the dimension of some relevant operator $\mathcal{O}$ and Λ is a mass scale. Hence the full string path integral (1.2) generates additional contributions to the worldsheet effective action of the form

$$S_T = -\int d^2\sigma \; \mu f(X^0) \; \mathcal{O}_\Delta(X_\perp, \Omega) + \int d^2\sigma \Phi(X^0) R^{(2)} + \text{fermions} \,, \tag{1.3}$$

where f has dimension $2 - \Delta$ in the unperturbed sigma model (1.2). We will henceforth refer to such deformations as "tachyons"; in the simplest case of an S^1 spatial component the corresponding mode is a standard winding tachyon. Let us discuss the big bang case for definiteness: the system becomes weakly curved in the future (large positive X^0) and goes singular at some finite value of X^0 in the past. The contribution (1.3) goes to zero as $X^0 \to +\infty$ since the sigma model is weakly coupled there. So at its onset the coefficient f increases as X^0 decreases; i.e. the effects of the term (1.3) increase as we go back in time in the direction of the would-be big bang singularity of the GR solution (1.1). In the simple case we will study in detail in section 3 below, a term of the form (1.3) will arise from winding tachyon condensation, and the operator f will be of the form $e^{-\kappa X^0}$ for real positive κ in the big bang case.

This growth of (1.3) as we approach the singularity contrasts to the suppression of the original sigma model kinetic terms from the metric (1.2). In the Minkowski path integral, the growth of the term (1.3) serves to suppress fluctuations of the fields. This provides a

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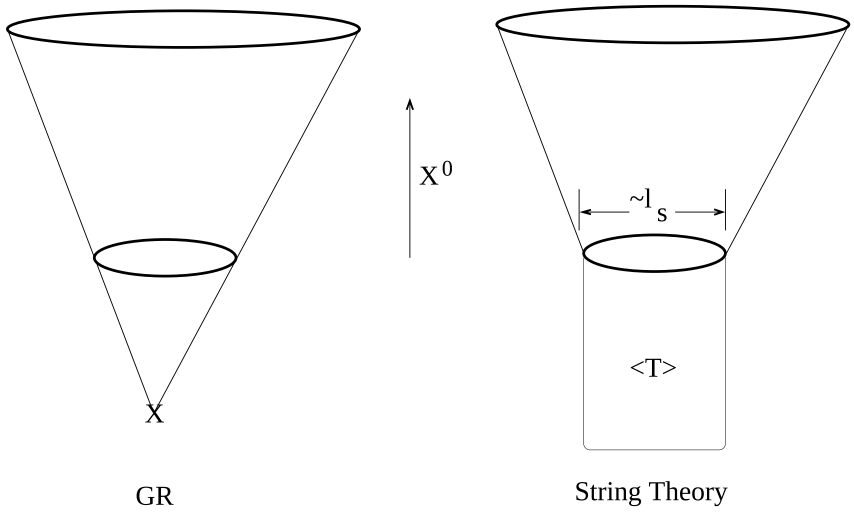

Figure 1: In string theory (with string length scale l_s), a tachyon condensate phase replaces a spacelike singularity that would have been present at the level of general relativity.

possibility of curing — via perturbative string effects — the singular amplitudes predicted by a naive extrapolation of GR. We will see this occur explicitly in the examples we will study in detail in section 3.

Because of the mass gap in the matter sector and the effect of the deformation S_T on the spacetime mass spectrum, the condensation of tachyons has long been heuristically argued to lift the string states and lead to a phase of "nothing" [9 – 14]. In the examples [1] where conical singularities resolve into flat space, this is borne out in detail, as the tip of the cone disappears in the region of tachyon condensation; a similar phenomenon was found for localized winding tachyons in [3]. In the present work, we will use methods from Liouville theory (for a review see [15 – 17]). We employ and extend the methods of [18 – 25] to perform systematic string theoretic calculations of amplitudes exhibiting this effect in our temporally but not spatially localized case. We study a particular vacuum state, analogous to the Euclidean vacuum. In this vacuum the support of string theoretic amplitudes is restricted to the bulk region of spacetime in a way that we can derive from the zero mode integral of X^0 in the worldsheet path integral.

As discussed above, the metric coefficient $G_{\Omega\Omega} = R(X^0)^2$ in the worldsheet action $S_0 + S_T$ (1.2) (1.3) goes to zero at finite X^0. In the models we consider below we will set up the system such that the approach to this singularity is parameterically slower than the timescale for the relevant term S_T to become important and lift the closed string degrees of freedom. This will avoid strong dilaton effects as well as effects of the shrinking space, and provides a mechanism which is inherently perturbative in g_s. The amplitudes will exhibit limited support in the spacetime, contributing only in the bulk region away from the epoch when the couplings become important. This is similar to the situation in spacelike Liouville theory, where similar strong coupling effects are avoided by the presence of the Liouville wall, and similar computational methods apply, though the physical mechanism for suppressing amplitudes is different in the two cases. Hence a self-consistent perturbative analysis is available. Relatedly, black hole formation of the sort found in [26, 27] is evaded here: the tachyon lifts the degrees of freedom of the system before the planckian regime is reached.

The observables of this theory are correlation functions of integrated vertex operators computed by the worldsheet path integral with semiclassical action $S_0 + S_T$ (1.2) (1.3); let us now discuss their spacetime interpretation. As in Liouville theory, the form of these operators is known in the weakly curved bulk region where there is no tachyon condensate ($X^0 \to \infty$ in the big bang case); there they asymptote in locally flat coordinates to operators of the form

$$V_{\vec{k},n} \to e^{i\vec{k}\cdot\vec{X}} e^{i\omega(\vec{k},n)X^0} \hat{V}_n \qquad \text{as} \qquad X^0 \to \infty\,, \tag{1.4}$$

where n labels the string state with mass m_n coming from oscillator excitations created by $\hat{V}_n$, $\vec{k}$ its spatial momentum, and $\omega^2 = \vec{k}^2 + m_n^2$.[2] Integrated correlation functions of these operators have the interpretation as components of the state of the strings in the bulk region of spacetime in a basis of multiple free string modes. In our example below, we will focus on a vacuum with no excitations above the tachyon condensate in the would-be singular region, and compute the resulting state of perturbative strings in the bulk region. This is a string-theoretic analogue of the Hartle-Hawking State (equivalently, the euclidean vacuum) on our time-dependent background.

We will treat the condensing tachyon in string perturbation theory. We obtain a self-consistent analysis at weak string coupling, in systems with bulk supersymmetry, and with supersymmetry breaking near the would-be singular region universe and inside black holes. Other interesting recent work on perturbative closed string calculations in time dependent backgrounds includes [27, 28]. In our case the tachyon condensation, related to the supersymmetry breaking of the time dependent background, plays a crucial role, in a way anticipated in [6].

It would be interesting to relate our analysis to other approaches based on non-perturbative formulations of the theory [29 − 31]. These approaches may provide a complete nonperturbative dual formulation of observables in spacetimes with singularities at the level of GR. On the other hand, the dictionary between the two sides is sometimes rather indirect as applied to approximately local processes on the gravity side. A useful feature of the current approach is that the tachyon condensation provides a direct gravity-side mechanism for quelling the singularity. It would be interesting to see how this information is encoded in the various dual descriptions.

Finally, analogously to the case of open string tachyons (for a review, see [32]), closed string tachyons may be a subject well-studied via closed string field theory; a candidate "nothing" state obtained from bosonic closed string bulk tachyon condensation was recently presented in [33]. It is clear (as we will review as we go) that the physics of the tachyon

[2] Although we do not know the form of these operators in the regime where the corrections (1.3) become important, we do know their conformal dimensions by virtue of their form in the bulk region of the spacetime. This is as in Liouville theory, where one knows the operators and the stress tensor away from the Liouville wall, and hence the spectrum of dimensions. And as in Liouville theory, an important question which we will address is where the amplitudes built from these operators have their support.

condensate is stringy — low energy effective field theory is not sufficient. In the setup we consider here, perturbative methods using techniques from Liouville theory will suffice, but in more general situations the off shell methods of string field theory may be required.

In the next two sections we set up and analyze a class of realizations of the mechanism. In section 4 we describe the generalization to positive spatial curvature, which is velocity-dominated. Philosophy-dominated comments are restricted to the concluding section.

2. Examples with winding tachyons

In this section, we will introduce the simplest backgrounds we will study; those with flat spatial slices which expand at a tunable rate. We will start with an example pertaining to 2+1 dimensional black holes (reducing to the 1+1 dimensional Milne spacetime inside), and then generalize to higher dimensional flat FRW cosmology with topologically nontrivial spatial slices and radiation.

2.1 The Milne spacetime

Consider the Milne spacetime described by the metric

$$ds^2 = -(dx^0)^2 + v^2(x^0)^2 d\Omega^2 + d\vec{x}^2 \,. \tag{2.1}$$

For $x^0 > 0$, this solution describes a growing S^1 along the Ω direction. At $x^0 = 0$ there is a spacelike big bang singularity, and general relativity breaks down. The evolution from $x^0 = -\infty$ to $x^0 = 0$ similarly describes an evolution toward a big crunch singularity. This geometry appears inside $2+1$ dimensional black holes, BTZ black holes in AdS_3.[3] We will show that for a wide class of string theories, the spacelike big bang or big crunch singularity (2.1) is evaded — the regime $|vx^0| < l_s$ is replaced by a phase of tachyon condensate.

In particular, to avoid bulk tachyons, consider type II, type I or heterotic string theory on the spacetime (2.1). Take antiperiodic boundary conditions around the Ω circle for spacetime fermions. Further consider the regime of parameters where $v \ll 1$. In addition to providing the control we will require, the last two conditions correspond to those appropriate for small BTZ black holes which can form naturally from excitations in pure AdS_3 (which has antiperiodic boundary conditions for fermions around the contractible spatial circle surrounding the origin).

With these specifications, we can determine with control the spectrum of string theory on the spacetime (2.1) for $x^0 \neq 0$. In the regime

$$v^2(x^0)^2 \leq l_s^2 \tag{2.2}$$

a closed string winding mode becomes tachyonic and hence important to the dynamics. The regime $v|x^0| \leq l_s$ of the singularity in (2.1) is replaced by a phase of tachyon condensate. This offers a concrete avenue toward resolving a spacelike singularity in string theory, and a corresponding notion of how time can begin or end.

[3] "Whisker" regions with closed timelike curves also appear in the maximally extended spacetime; our methods here will also have the effect of excising these regions, as obtained in other examples in [4].

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This in itself is worth emphasizing. The problem of bulk tachyon condensation is often motivated by the question of the vacuum structure of string theory. The present considerations provide an independent motivation for pursuing the physics of closed string tachyon condensation: it appears crucially in a string-corrected spacelike singularity. In our system here there is no tachyonic mode in the bulk of spacetime: for a semiinfinite range of time the system is perturbatively stable. That is, the tachyon phase is localized in time. As we will see, this provides significant control over the problem even though the condensation is not also localized in space.

2.2 Flat FRW with topology

Next let us set up a somewhat more realistic case which shares the essential features of the above example. Consider flat-sliced FRW cosmology with bulk metric

$$ds^2 = -(dx^0)^2 + a^2(x^0)d\vec{x}^2 + ds_\perp^2 \tag{2.3}$$

with $\vec{x}$ a 3-dimensional spatial vector and $ds_\perp^2$ describing the extra dimensions. Let us consider some periodicity in the spatial directions $\vec{x}$: $\vec{x} \equiv \vec{x} + \vec{L}_I$; e.g. letting I run from 1 to 3 produces a spatial torus (for simplicity let us take a rectangular torus). In real cosmology, such topology could well exist at sufficiently large scales (most generically well outside our horizon today due to inflation), but if present would play a role in the far past in the epoch of the would-be big bang singularity. (See e.g. [34] for one recent discussion of spatial topology.). Let us study the above system in the presence of a stress-energy source. For definiteness, consider a homogeneous bath of radiation. The Friedmann equation implies

$$a(x_0) = a_0\sqrt{x^0 - t_0}\,, \tag{2.4}$$

where the coefficient a_0 can be tuned by dialing the amount of radiation.

In particular, as in the above example (2.2), we can choose the radiation density and hence a_0 so as to obtain a slow expansion of the toroidal radii $R \sim La(x^0)$ as the smallest radius passes through the string scale. Again considering antiperiodic boundary conditions for fermions along one or more of the 1-cycles of the torus, we then obtain in a controlled way a winding tachyon in the system as the radius $R \sim La(x^0)$ of a circle passes below the string scale. The would-be big bang singularity is again replaced by a tachyon condensate phase, whose consequences we will analyze in detail in the next section.

3. Examples with winding tachyons: some basic computations of observables

In this section, we develop a systematic computational scheme to compute physical observables in this system, assess back reaction, and test and make more precise the proposition that tachyon condensation lifts closed string excitations (leading to a phase we will refer to as a nothing state).

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3.1 Wick rotation

Let us start by defining the path integral via appropriate Wick rotation. In its original lorentzian signature form, the tachyon term appears to increase the oscillations of the integrand, hence suppressing contributions in the region of the tachyon condensate. As is standard in quantum field theory, we will perform a Wick rotation to render the path integral manifestly convergent (up to, as we will see, divergences at exceptional momenta expected from the bulk S-matrix point of view). The path integral in conformal gauge includes an integral over the target space time variable X^0, which has a negative kinetic term in the worldsheet theory. Because this field also appears necessarily in the tachyon interaction term (which is proportional to $e^{-\kappa X^0}$, specializing to the big bang case), we will find it convenient to Wick rotate the worldsheet theory to directly obtain real positive kinetic terms for X^0 without rotating the contour for X^0 integration; this will entail rotating the contours for the spatial target space coordinates as well as continuing μ in a way we will specify. (Alternatively one could rotate X^0 as is standardly done in the free theory, and continue in κ at the same time, as in [20].)

Prelude: worldline quantum field theory. Before turning to the full string path integral, let us briefly describe a much simpler analogue of our system which arises in the worldline description of quantum field theory, as emphasized in [35]. Consider a relativistic particle action

$$S = \int d\tau \left(-(\partial_\tau X^0)^2 + (\partial_\tau \vec{X})^2 - (m_0^2 + \mu^2 e^{-2\kappa X^0}) \right), \tag{3.1}$$

where we have included a time-dependent mass squared term $m^2(X^0) = m_0^2 + \mu^2 e^{-2\kappa X^0}$.

For $\mu^2 > 0$, this theory describes a particle with a time-dependent positive mass-squared that increases exponentially in the past $X^0 \to -\infty$. The potential term in the relativistic worldline action leads to a lifting of particles in the region where it becomes important. If one starts with none of these massive modes excited in the past, then the future state gets populated due to the time dependent mass. The Bogoliubov coefficient $\beta_{\vec{k}}$ describing mixing of positive and negative frequency modes has magnitude $e^{-\pi\omega/\kappa}$ with $\omega = \sqrt{\vec{k}^2 + m_0^2}$ the frequency of the particle modes in the region $X^0 \to +\infty$. We will find similar features in our string theoretic examples, where the phase in which states are lifted replaces a spacelike singularity.

For $\mu^2 < 0$, this theory describes a system with time-dependent negative mass squared. A particle with positive mass squared in the far future becomes tachyonic in the far past. One could formally start again with no excitations in the past, but this would be unnatural as the tachyonic modes there would condense.

In lorentzian signature, the worldline path integral is

$$\int [dX]\, e^{iS} . \tag{3.2}$$

If we continue $\tau \equiv e^{i\gamma}\tau_\gamma$ and $\vec{X} \equiv e^{i\gamma}\vec{X}_\gamma$, taking γ continuously from 0 to $\pi/2$, we obtain a path integral

$$\int [dX_E] \, \exp\left[-\int d\tau_E \left((\partial_{\tau_E}X^0)^2 + (\partial_{\tau_E}\vec{X})^2 + \mu^2 e^{2\kappa X^0}\right)\right]. \tag{3.3}$$

Ambiguities in defining the X^0 integral correspond to choices of vacuum state. In order to obtain a convergent path integral, we can continue $\mu^2 \to -\mu^2$ (i.e. $\mu \to e^{-i\gamma}\mu$) as we do the above Wick and contour rotation, compute the amplitudes, and then continue back. That is, our computation is related to one in a purely spacelike target space via a reflection of the potential term in the worldline theory.

This reflection appears also in the direct spacetime analysis of particle production in field theory with time dependent mass. The Heisenberg equation of motion satisfied by the Heisenberg picture fields in spacetime takes the form of a Schrödinger problem for each $\vec{k}$ mode. The effective potential in the Schrödinger problem is $U_{eff} = -(m^2(X^0) + \vec{k}^2)$. This leads to highly oscillating mode solutions as $X^0 \to -\infty$, reflecting the exponentially increasing mass in the far past.

With this warmup, let us now turn to the case of string theory with a tachyon condensate, which has a worldsheet potential term analogous to the mass squared term in the field theory case. We will study both the heterotic and type II theories on our background.

Heterotic theory. In the RNS description of the heterotic string, the worldsheet theory has local (0,1) supersymmetry. This case is in some ways the simplest for studying closed string tachyons using the string worldsheet description – unlike the bosonic theory, there is no tachyon in the bulk region where the S^1 is large; unlike the type II theory the worldsheet bosonic potential is automatically nonnegative classically (as in the open superstring theory).

There is a choice of discrete torsion in the heterotic theory on a space with shrinking Scherk-Schwarz circle. The background can be regarded as a $\mathbb{Z}_2$ orbifold of a circle by a shift halfway around combined with an action of $(-1)^F$ where F is the spacetime fermion number. Combining this $\mathbb{Z}_2$ with that of the left-moving fermions (in say the SO(32) Heterotic theory) yields two independent choices of action of the left moving GSO on the states of the Scherk-Schwarz twisted sector. A standard choice, giving rise to a Hagedorn tachyon, is to act trivially on the Scherk-Schwarz twisted sector; this yields a twisted sector tachyon made from momentum and winding modes [9].

This would also be the most natural choice for us in some sense, since the usual spacelike singularities in cosmology and inside black holes are a priori independent of Yang-Mills degrees of freedom. However because the winding+momentum twisted tachyon in the above case is a nonlocal operator on the worldsheet in both T-duality frames, we will make here a technically simpler choice. Namely, we can choose the discrete torsion such that the left-moving GSO projection acts nontrivially on the states of the twisted sector, yielding a twisted tachyon created by a left moving fermion and a winding operator.

In the heterotic theory we have target space coordinates given by (0,1) scalar superfields $\mathcal{X}^\mu = X^\mu + \theta^+ \psi_+^\mu$ and left moving fermion superfields $\Psi_-^a = \psi_-^a + \theta^+ F^a$ containing auxiliary fields F^a. In terms of these fields we have a lorentzian signature path integral

$$G(\{V_n\}) \equiv \int [d\mathcal{X}][d\Psi_-][d(\text{ghosts})]d(\text{moduli})\; e^{iS} \prod_n \Big(i \int d\sigma d\tau V_n[\mathcal{X}] \Big) \,, \tag{3.4}$$

where the semiclassical action is

$$\begin{aligned} iS = i \int d\sigma d\tau d\theta^+ \Big(& D_{\theta^+} \mathcal{X}^\mu \partial_- \mathcal{X}^\nu G_{\mu\nu}(\mathcal{X}) - \mu \Psi_- : e^{-\kappa \mathcal{X}^0} \cos(w\tilde{\Omega}) : \\ & + \Psi_-^a D_{\theta^+} \Psi_-^a + (\text{dilaton}) \Big) + iS_{\text{ghost}} \end{aligned} \tag{3.5}$$

and $V_n[\mathcal{X}]$ are vertex operator insertions. Here $\tilde{\Omega}$ is the T-dual of the coordinate Ω on the smallest circle in the space (let us consider for genericity a rectangular torus, whose smallest cycle will play a leading role in the dynamics); $\cos w\tilde{\Omega}$ is the winding operator for strings wrapped around the Ω direction.

The case of no insertions corresponds to the vacuum amplitude Z. The fluctuations of the worldsheet fields in (3.4) generate corrections to the action (3.5); for example, the term proportional to μ coming from the tachyon condensate is marginal but not exactly marginal. This is similar to the form of the Liouville wall in Liouville field theory, which is a priori only semiclassically given by a pure exponential.

Similarly, the form of the vertex operators is known semiclassically. Because the bulk region of the geometry (2.1) is approximately flat space, we may identify the V_n with operators of the form

$$V_{\vec{k},n} \to e^{i\vec{k}\cdot\vec{\mathcal{X}}} e^{i\omega(\vec{k},n)\mathcal{X}^0} \hat{V}_n \qquad \text{as} \qquad X^0 \to \infty \,, \tag{3.6}$$

where as in (1.4) we have pulled out the oscillator and ghost contributions into $\hat{V}$.

Finally, at the semiclassical level the dilaton is also known: it goes to a constant

$$\Phi \to \Phi_0 \qquad \text{as} \qquad X^0 \to +\infty \,. \tag{3.7}$$

In particular, the tachyon vertex operator in (3.5) is semiclassically marginal without an additional dilaton contribution (though not exactly marginal) and the metric terms solve Einstein's equations. The path integral over fluctuations of the fields will generate corrections to these semiclassical statements (3.5)–(3.7).

The basic physical effect of the tachyon condensate is the oscillations it induces in the path integral, suggesting a suppression of contributions rather than a divergence in the would-be crunch region. For some computations, it is useful to Wick rotate in evaluating the path integral (though as we will see below some computations such as the 1-loop partition function could be evaluated more directly).

Let us Wick rotate the worldsheet time coordinate τ, the spatial target space coordinates $\vec{\mathcal{X}}(\sigma,\tau)$ (including $\tilde{\Omega}$), and the parameters μ and $\vec{k}$ by

$$\tau \equiv e^{i\gamma}\tau_\gamma \qquad \vec{\mathcal{X}} \equiv e^{i\gamma}\vec{\mathcal{X}}_\gamma \qquad \mu = e^{-i\gamma}\mu_\gamma \qquad \vec{k} = e^{-i\gamma}\vec{k}_\gamma \,, \tag{3.8}$$

where γ is a phase which we will rotate from 0 to $\pi/2$. This produces a Euclidean path integral for the worldsheet theory (where we label the quantities rotated to $\gamma = \pi/2$ by a subscript E)

$$G(\{V_n\}) \equiv \int [d\vec{\mathcal{X}}_E][d\mathcal{X}^0][d\Psi_-][d(\text{ghosts})]d(\text{moduli})\ e^{-S_E} \prod_n \int (-1) d\sigma d\tau_E V_{n,-i\vec{k}_E}[\mathcal{X}^0, i\vec{\mathcal{X}}_E] \tag{3.9}$$

with euclidean action

$$\begin{aligned} S_E = \int d\sigma d\tau_E d\theta^+ \Big(& D_{\theta^+}\mathcal{X}^0 \partial_- \mathcal{X}^0 + v^2 (\mathcal{X}^0)^2 D_{\theta^+}\tilde{\Omega}_E \partial_- \tilde{\Omega}_E + G_{ij} D_{\theta^+} \mathcal{X}^i_{\perp,E} \partial_- \mathcal{X}^j_{\perp,E} - \\ & - i\mu_E e^{-\kappa \mathcal{X}^0} \cosh(w\tilde{\Omega}_E) + \\ & + \Psi^a_- D_{\theta^+} \Psi^a_- + (\text{dilaton}) \Big) + S_E(\text{ghost})\,. \end{aligned} \tag{3.10}$$

Here $\vec{\mathcal{X}}_E \equiv (\Omega_E, \vec{\mathcal{X}}_{\perp E})$ refers to the worldsheet superfields corresponding to the spatial target space coordinates, and we have plugged in the spacetime metric (2.1).

The procedure (3.8) alone yields a formal path integral whose tachyon term is not marginal in the Euclidean theory. In order to retain semiclassical conformal invariance, we could include in (3.8) a linear dilaton coefficient: $Q_\gamma \equiv \gamma(2/\pi)Q$ where Q corresponds to the coefficient of the linear dilaton which renders the tachyon vertex operator marginal at leading order. This is similar to part of the procedure followed in e.g. [20]. Alternatively we can analyze the theory for $Q = 0$, in which case the intermediate steps in the analysis involve path integral computations in a nonconformal Euclidean worldsheet theory, obtained by analytic continuation in κ from a conformal Euclidean theory. As we will see, for our computations of the vacuum energy and of particle production, both procedures yield the same result.

In the specification of operators in the bulk region we have neglected terms proportional to the slow velocity v by which the circles shrink in the bulk metric (2.3) (2.1). Relatedly, μ could depend weakly on the other spatial directions $\vec{X}_\perp$; we will ignore this for the purposes of the current discussion though it is simple to incorporate.

In fact the small velocity approximation will play an important role more generally in our analysis of the approach to the singularity. As it stands, the path integral (3.9) (3.10) does not extend over all values of X^0: the metric term $G_{\Omega\Omega}$ of classical GR goes to zero in finite X^0 in the past. As we will see, for a constant radius circle of size L, a winding tachyon condensate will produce a truncation of the support of amplitudes to a range of X^0 of order $(\ln \mu)/\kappa$ in the region of the condensate. We can arrange the parameters in our worldsheet CFT such that the velocity is sufficiently small that this range of time is far smaller than the time it takes to reach the singularity starting from a circle of size L. The basic idea is that the effective Newton constant does get large as the space shrinks, but the effects of the tachyon kick in first. Namely, consider a winding tachyon which turns on when the circle size is L. The corresponding value of κ is $\kappa = \sqrt{1 - (L/l_s)^2}$. Set v such that

$$\frac{L}{v} \gg -\frac{\ln \mu}{\kappa}\,. \tag{3.11}$$

This specification, combined with the $(\ln\mu)/\kappa$ truncation of the amplitudes' support in the X^0 direction to be derived below, yields a self-consistent perturbative string analysis. Note that the worldsheet potential term in the heterotic theory is classically always non-negative.

From this well-defined path integral (3.9) (3.10) we will obtain the μ dependence of amplitudes using methods developed for Liouville theory which also apply to our theory. This will enable us to read off the effect of the tachyon on the support of amplitudes, and will determine the spectrum of particles produced due to the time dependence of the tachyon background.

Type II theory. Let us next briefly include the type II version of the above formulas. In the type II theory, we have (1,1) scalar superfields $\mathcal{X}^\mu = X^\mu + \theta^+\psi_+^\mu + \theta^-\psi_-^\mu + \theta^+\theta^- F^\mu$. In terms of these, we have a lorentzian signature path integral

$$G(\{V_n\}) \equiv \int [d\mathcal{X}][d(\text{ghosts})]d(\text{moduli})\; e^{iS} \prod_n \left(i \int d\sigma d\tau V_n[\mathcal{X}] \right), \tag{3.12}$$

where the semiclassical action is

$$iS = i\int d\sigma d\tau d\theta^+ d\theta^- \Big(D_{\theta^+}\mathcal{X}^\mu D_{\theta^-}\mathcal{X}^\nu G_{\mu\nu}(\mathcal{X}) - \mu : e^{-\kappa\mathcal{X}^0}\cos(w\tilde{\Omega}) : \\ + (\text{dilaton}) \Big) + iS_{\text{ghost}} \tag{3.13}$$

and $V_n[\mathcal{X}]$ are vertex operator insertions. As in the heterotic case, the form of the vertex operators is known in the flat space region to be of the form (3.6). The dilaton is (3.7).

Let us Wick rotate the worldsheet time coordinate τ, the spatial target space coordinates $\vec{X}(\sigma,\tau)$ (including $\tilde{\Omega}$), and the parameters μ and $\vec{k}$ by

$$\tau \equiv e^{i\gamma}\tau_\gamma \qquad \vec{\mathcal{X}} \equiv e^{i\gamma}\vec{\mathcal{X}}_\gamma \qquad \mu = e^{-i\gamma}\mu_\gamma \qquad \vec{k} = e^{-i\gamma}\vec{k}_\gamma \tag{3.14}$$

where γ is a phase which we will rotate from 0 to $\pi/2$. This produces a euclidean path integral for the worldsheet theory (where we label the quantities rotated to $\gamma = \pi/2$ by a subscript E)

$$G(\{V_n\}) \equiv \int [d\vec{\mathcal{X}}_E][d\mathcal{X}^0][d(\text{ghosts})]d(\text{moduli})\; e^{-S_E} \prod_n \int (-) d\sigma d\tau_E V_{n,-i\vec{k}_E}[\mathcal{X}^0, i\vec{\mathcal{X}}_E] \tag{3.15}$$

with euclidean action

$$S_E = \int d\sigma d\tau_E d\theta^+ d\theta^- \Big(D_{\theta^+}\mathcal{X}^0 D_{\theta^-}\mathcal{X}^0 + v^2(\mathcal{X}^0)^2 D_{\theta^+}\tilde{\Omega} D_{\theta^-}\tilde{\Omega} + G_{ij} D_{\theta^+}\mathcal{X}^i_{\perp,E} D_{\theta^-}\mathcal{X}^j_{\perp,E} - \\ - i\mu_E e^{-\kappa\mathcal{X}^0}\cosh(w\tilde{\Omega}_E) + (\text{dilaton}) \Big) + S_E(\text{ghost})\,. \tag{3.16}$$

Again, as discussed below (3.10), we could include a γ-dependent shift of the linear dilaton coefficient (which vanishes in our physical critical-dimension lorentzian bulk theory) if we wish to maintain semiclassical conformal invariance in the intermediate (rotated) steps of the calculations.

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3.2 Vacuum amplitude and back reaction

In this subsection we will present computations exhibiting the effect we advertised above that the amplitudes will be limited in their support to the weakly-coupled bulk. Let us start with the vacuum amplitude. At one loop, this is defined by the amplitude (3.15) with no vertex operator insertions, evaluated on a genus one worldsheet; let us call this amplitude Z_1. In a time dependent background, one must specify the vacuum in which the amplitudes are defined (for example, one definition of the S matrix would involve in-vacuum to out-vacuum amplitudes). We will return to the question of the vacuum after computing the first quantized path integral defined above at this 1-loop order.

In the bulk, this quantity describes a trace over spacetime single-particle states. In our case, the integral over the zero mode of X^0 will work differently than in flat space, and we will determine from this the support of the amplitudes as well as the quantum corrections to the stress energy in spacetime. In particular, as in Liouville field theory, we will find this amplitude to be supported only in the bulk region where the tachyon condensate is small. This supports the interpretation of the tachyon condensate as lifting the closed string degrees of freedom. Further, with our asymptotic supersymmetry in the bulk region this also provides a useful bound on the back reaction in the model. Finally, the imaginary part of the amplitude will provide information about the vacuum with respect to which the amplitude is being computed from the spacetime point of view.

Following [36, 15], let us compute first the quantity $\partial Z_1/\partial\mu$ and perform the path integral by doing the integral over the X^0 zero modes first. That is, decompose

$$X^0 \equiv X^0_0 + \hat{X}^0(\sigma,\tau_E)\,, \tag{3.17}$$

where $\hat{X}^0$ contains the nonzero mode dependence on the worldsheet coordinates σ,τ_E.[4] The path integral measure then decomposes as $[dX^0] = dX^0_0[d\hat{X}^0]$. We obtain for heterotic and type II respectively

$$\begin{aligned}
\frac{\partial Z_1^{(Het)}}{\partial\mu_E} &= \int [d\vec{\mathcal{X}}_E][d\Psi_-][d(\text{ghosts})]d(\text{moduli})[d\hat{\mathcal{X}}^0]dX^0_0 \times \\
&\quad\times\left(-\int d\sigma d\tau_E d\theta^+\Psi_- e^{-\kappa\mathcal{X}^0} i\cosh(w\tilde{\Omega}_E)\right)e^{-S_E} &(3.18)\\
\frac{\partial Z_1^{(II)}}{\partial\mu_E} &= \int [d\vec{\mathcal{X}}_E][d(\text{ghosts})]d(\text{moduli})[d\hat{\mathcal{X}}^0]dX^0_0 \times \\
&\quad\times\left(-\int d\sigma d\tau_E d\theta^+ d\theta^- e^{-\kappa\mathcal{X}^0} i\cosh(w\tilde{\Omega}_E)\right)e^{-S_E}\,. &(3.19)
\end{aligned}$$

Decomposing $e^{-\kappa\mathcal{X}^0} = e^{-\kappa X^0_0}e^{-\kappa\hat{\mathcal{X}}^0}$, we can change variables in the zero mode integral to $y \equiv e^{-\kappa X^0_0}$ and integrate from $y=0$ to $y=\infty$ as X^0_0 ranges from ∞ to $-\infty$.[5] For each

[4]The reader should be grateful that we are suppressing the atomic number and baryon number indices on ${}^0_0X^0_0$.

[5]Note that the support of the integrand is negligible in the added region $X^0_0 \in [-\infty, 0]$.

point in worldsheet field space, the zero mode integral is of the form

$$\int_0^\infty dy\ e^{-Cy} = \frac{1}{C}\,, \tag{3.20}$$

where the coefficient C is the nonzeromode part of the tachyon vertex operator in S_E, integrated over worldsheet superspace.

Integrating over $\theta^\pm$ produces a worldsheet potential term contributing to C. For regions of field space where C is positive, the integral (3.20) converges. For regions of negative C the equation (3.20) gives *a* formal definition of the function by analytic continuation. However, it is important to keep in mind the physical distinction between these two cases. As discussed above in the quantum field theory case, when C is positive this corresponds to a time dependent massing up of modes, while negative C corresponds to time dependent tachyonic masses. In the latter case, the formal definition (3.20) describes an analytic continuation of an interesting physical IR divergence.

In the heterotic theory, this coefficient C is nonnegative everywhere in field space for $\mu^2 > 0$, at least classically. Hence the computation (3.20) applies directly.

In the type II theory, this coefficient can become negative near particular points in $\tilde{\Omega}$ and $\vec{X}_\perp$. In regions where the potential is positive, (3.20) applies, and as we will see leads to a truncation of the support of the closed string states. However, in regions where C is negative, there are physical instabilities remaining. These localized instabilities we interpret as subcritical type 0 tachyons. In particular, in section 4.1 we will see using linear sigma model techniques that the GSO projection acts on the corresponding subcritical theory as in type 0.

This analysis yields

$$\frac{\partial Z_1}{\partial \mu_E} = -\frac{1}{\kappa \mu_E}\hat{Z}_1\,, \tag{3.21}$$

where $\hat{Z}_1$ is the partition function in the free theory (with no tachyon term in the action) and with no integral over the zero mode of X^0. Referring to the functional measure for the rest of the modes (including all fields) as $[d(\text{fields})]'$ this is

$$\hat{Z}_1 = \int [d(\text{fields})]'[d(\text{ghosts})]d(\text{moduli})\ e^{-\hat{S}_E}\,, \tag{3.22}$$

where $\hat{S}_E$ is the action ((3.10) and (3.16) respectively for heterotic and type II) with $\mu = 0$. Finally, integrating with respect to μ yields the result for the 1-loop partition function

$$Z_1 = -\frac{\ln(\mu_E/\mu_*)}{\kappa}\hat{Z}_1\,. \tag{3.23}$$

Here $\mu_* = e^{\kappa X^0_*}$ where X^0_* is an IR cutoff on the X^0 field space in the free-field region. As discussed above, this is valid for regions of the worldsheet field space where the potential is positive, which is always true in the Heterotic case and true for most contributions in type II.

It is worth remarking here that this result follows simply from the exponential form of the potential in the tachyon vertex operator. The above derivation formally goes through whether rotated back to target space Minkowski signature or not, (since the zeromode does not appear in the kinetic term), reducing the amplitude to a tachyon-free computation $\hat{Z}$. Relatedly it is insensitive to the issue of rotating in Q as discussed below (3.10).

To interpret this result, recall that in a background of d-dimensional flat space, the partition function scales like the volume of spacetime: integration over the zero modes of the X^μ fields yields the factor $\delta^{(d)}(0) = V_d = V_0 V_{d-1}$ where V_{d-1} is the volume of space and V_0 is the volume of the time direction. In our present case (3.23), the spatial extensivity reflected in the factor V_{d-1} is still present. But the volume of time V_0 has been truncated to $-\frac{1}{\kappa}\ln\mu/\mu_*$. This corresponds to the range of X^0 where the tachyon is absent. Again, this is similar to the situation in spacelike Liouville field theory [36], where the Liouville wall cuts off the support of the partition function.

This result has several implications. First, it provides a concrete verification that the string states are lifted in the tachyon phase, for positive worldsheet potential, supporting the interpretation of this phase as a theory of nothing. Combined with the specification (3.11) this result justifies the use of the worldsheet path integral with metric coefficients going to zero in finite time, as the amplitudes are not supported in this region. Note that in particular all states are lifted — the would-be tachyon and graviton fluctuations are absent and hence back reaction is suppressed. As emphasized above, we are computing in a particular state analogous to the euclidean or Hartle-Hawking vacuum, with no excitations in the far past. Putative vacua with excitations in the far past will be analyzed separately [37]. The analagous context of quantum field theory with exponentially growing mass again suggests that the string states will become lifted (effectively go to infinite mass) in an appropriate sense; moreover an interesting BRST anomaly appears which may limit the consistency of such vacua in the string theory context. (See [38] for some discussion of the issue in field theory).

Second, it indicates that the 1-loop vacuum energy is only supported in the bulk region of the spacetime. Because the asymptotic bulk region $X^0 \to \infty$ is weakly coupled and weakly curved (in fact approximately supersymmetric), this means that back reaction is restricted to the intermediate region where the tachyon T is of order 1.

Third, the imaginary part of the 1-loop partition function is significant and will provide an important check on the consistency of our computations. Recall that the analytic continuation (3.14) included a rotation $\mu = e^{-i\pi/2}\mu_E$. This means that for real values of our original parameter μ, the partition function has an imaginary part:

$$Z_1 = \left(-\frac{1}{\kappa}\ln\frac{\mu}{\mu_*} - i\frac{\pi}{2\kappa}\right)\hat{Z}_1 \,. \tag{3.24}$$

Note that the IR cutoff X^0_* is always real in our prescription. We will interpret this as indicating that the system is in a pure state with thermal occupation numbers corresponding to a temperature κ/π, a result which will also follow from an analysis of the 2 point amplitudes at genus zero. Let us return to this interpretation after explaining those computations in the next subsection.

In general, it would be interesting to unpack the 1-loop amplitudes in more detail, to follow the fate of the various closed string states and D-branes in our background. An important aspect of this is mode mixing induced by the tachyon operator: the oscillator modes in the bulk generally mix under the action of the tachyon term in the region where it is substantial.

It might be possible to analyze this using the ideas in [39]. In the type II case, a similar compuational technique to the one we have described above applies to the amplitudes of open strings in this background, for example the 1-loop open string partition function. The closed string channel of such amplitudes describes the response of the would-be graviton and other closed string modes to D-brane sources. It is necessary to specify consistently the boundary conditions defining the D-branes in this background, but it seems likely that the X_0^0 integral will again reveal that these amplitudes are shut off in the tachyon phase. We note that in the spacelike Liouville theory, the ZZ-brane [40] is localized under the tachyon barrier, and has a paucity of degrees of freedom. It cannot move; basically it can only decay. It would be very interesting to understand the conformal boundary states in the timelike case.

3.3 2-point function, particle production, and Euclidean State

Let us now include vertex operator insertions. The μ-dependence of amplitudes can be determined by a similar technique to that above. We analyze the derivative of the correlation function (3.15)[6] with respect to μ_E by doing the integral over X^0's zero mode X_0^0 first. From that we can determine its dependence on μ_E, and finally use (3.14) to determine its dependence on μ.

This is similar to the above computation of the partition function, except now the integral over $y = e^{-\kappa X_0^0}$ (which gave (3.20) in the case of the partition function) is of the form

$$\frac{\partial G(\{V_{n,-i\vec{k}_E}\})}{\partial \mu_E} = \int [d\hat{\mathcal{X}}^0][d\vec{\mathcal{X}}][d(\text{ghost})] \int dy\ y^{-\sum_n i\frac{\omega_n(\vec{k}_n)}{\kappa}} e^{-Cy} e^{-\hat{S}_E} \frac{C}{\mu_E}\,, \tag{3.25}$$

where C is defined after (3.20). This yields a result for $G(\{V_{n,-i\vec{k}_E}\})$ proportional to

$$\mu_E^{i\sum_n \omega_n/\kappa} \tag{3.26}$$

times a complicated path integral over nonzero modes, which would be difficult to evaluate directly, but which is independent of μ_E. As discussed below (3.10), we could rotate in a way that introduces a linear dilaton in the euclidean continuation. This would shift the ω-dependent exponent by a term linear in Q, which would rotate back to zero when we rotate μ_E back to μ.

Fortunately, in the case of the 2-point function, we can use a simple aspect of the analytic continuation we used to define the path integral to determine the magnitude of the result. As explained for example in [41], the two-point function of two negative

[6] Note that as in LFT, we use the semiclassical form of the vertex operators and dilaton as well as of the the action.

frequency modes in the bulk is

$$G(\vec{k},n;\vec{k}',n') = \delta_{nn'}\delta(\vec{k}-\vec{k}')\frac{\beta_{\vec{k},n}}{\alpha_{\vec{k},n}}\,, \tag{3.27}$$

where $\alpha_{\vec{k},n}$ and $\beta_{\vec{k},n}$ are the Bogoliubov coefficients describing the mixing of positive and negative frequency modes. This is the timelike Liouville analogue of the reflection coefficients describing the mixing of positive and negative momentum modes bouncing off a spacelike Liouville wall.

In fact, this relation is precise here, and we can determine the magnitude $|\beta_\omega/\alpha_\omega|$ as follows. As we discussed above for the partition function, after performing the Euclidean path integral defined via the rotations (3.14), we must continue back $\mu \to -i\mu$ in order to obtain the amplitude of interest. The regions where the worldsheet potential is positive translate in the Euclidean path integral to a positive Liouville wall. For these regions, the Euclidean 2-point function is a reflection coefficient of magnitude 1. The continuation above (3.8) (3.14) in μ,

$$\mu = e^{-i\frac{\pi}{2}}\mu_E \tag{3.28}$$

therefore yields a 2-point function for the lorentzian theory of magnitude

$$\left|\frac{\beta_{\vec{k},n}}{\alpha_{\vec{k},n}}\right| = e^{-\omega(\vec{k},n)\pi/\kappa}\,. \tag{3.29}$$

Using the relations $|\alpha_\omega|^2 \mp |\beta_\omega|^2 = 1$ for bosonic and fermionic spacetime fields, and the fact that the number of particles produced $N_{\vec{k},n}$ is given by $|\beta_{\vec{k},n}|^2$, this result translates into a distribution of pairs of particles of a thermal form

$$N_{\vec{k},n} = \frac{1}{e^{2\pi\omega(\vec{k},n)/\kappa} \mp 1}\,. \tag{3.30}$$

This corresponds to a Boltzmann suppression of the distribution of pairs of particles (each pair having energy 2ω) by a temperature $T = \kappa/\pi$. This temperature can also be deduced from the imaginary part of the 1-loop partition function (3.24), providing a check on the calculations, as follows.

At the level of the genus zero two-point functions, the system is in a pure state whose phase information we have not computed, but whose number density is thermal. We would like to understand how the result (3.24) fits into this thermal interpretation. To explain this, for simplicity let us work in a field theory limit in the bulk, so that we can express the state in terms of the bulk Fock space of states built on the bulk vacuum $|\text{bulk}\rangle$ killed by the annihilation operators $a_{\vec{k}}$. Ignoring interactions, the bulk state is of the form of a squeezed state

$$|\Psi_0\rangle = \mathcal{N}\exp\left(\sum_{\vec{k}} e^{i\gamma_{\vec{k}}}e^{-\omega(\vec{k})\kappa/\pi}a^\dagger_{\vec{k}}a^\dagger_{-\vec{k}}\right)|\text{bulk}\rangle\,, \tag{3.31}$$

where $\mathcal{N}$ is a normalization factor and the $\gamma_{\vec{k}}$ are the phases arising in the two point function. Expanding the exponent, this is of the form

$$|\Psi_0\rangle = \mathcal{N}\sum_n e^{i\gamma_n}e^{-\frac{\beta_T}{2}E_n}|E_n\rangle \tag{3.32}$$

for some real phases γ_n, where n indexes the Fock space states and E_n the corresponding energies, and where $\beta_T = \pi/\kappa$ is the inverse temperature.

Switching back to the Schrodinger picture for simplicity, the bulk state can be thought of then as evolving as if the time evolution operator, normally $U(t,0) = T(\exp(-i\int_0^t dt'\, H(t')))$, is now $T(\exp(-i\int_C dt' H(t')))$ where the contour C runs from 0 to t along the real axis, and then runs vertically from t to $t - i\beta_T/2$.[7]

Now let us incorporate the 1-loop vacuum energy correction (3.24). This calculates the zero point energy contribution to the spacetime hamiltonian, integrated over time. As we have just seen, in a state with thermal occupation numbers, the effective hamiltonian evolution up to time t is on a time contour with an extra segment from t to $t - i\beta_T/2$. The result (3.24) arises from the bulk vacuum result via exactly such a shift, with $\beta_T = \pi/\kappa$ corresponding to a temperature $T = \kappa/\pi$. Namely, as discussed in section 3.3, the partition function is the summed zero-point energy in spacetime, times the volume of time: $Re(Z) = \Lambda V_{d-1} V_0$. Said differently, $Z = \int dt \Lambda V_{d-1}$ where t is the target-space time direction. The imaginary part of our amplitude (3.24) is obtained from the bulk vacuum by shifting the spacetime hamiltonian evolution by $\Lambda V_{d-1} X^0 \to \Lambda V_{d-1}(X^0 - i\frac{\pi}{2\kappa})$. So the same temperature arises in both the genus zero and the 1-loop corrected amplitudes in our prescription, providing a check.

Altogether, this yields the following simple result. Let us consider the big bang case, with the tachyon condensate turned on in the past. Modulo expected subcritical tachyons in the type II case, the closed string states are lifted in the far past (and in the type II case, we expect the subcritical tachyons to also condense and lift degrees of freedom). Start with no excitations above this tachyon background (perhaps a natural choice given the enormous effective masses in this region). The state in the bulk $X^0 \to \infty$ region has a thermal distribution of pairs of particles (3.30), with temperature κ/π. These pairs are created during the phase where the tachyon condensate is order one,[8] and hence the calculation is self-consistent if we tune the bare dilaton to weak coupling.

This choice of state is analogous to the Hartle-Hawking, or Euclidean, State in the theory of quantum fields on curved space, but it arises here in a perturbative string system via crucially stringy effects. In quantum field theory on curved space, the Euclidean vacuum is obtained by calculating Greens functions in the Euclidean continuation of the spacetime background (when it exists) and continuing them back to lorentzian signature. In our case, a similar continuation has been made, but here the Euclidean system is a spacelike Liouville field theory. The choice of vacuum (nothing excited above the tachyon condensate) is natural from the point of view of the spacelike continuation, as it involves excitations of modes with only one sign of frequency in the tachyon phase; these Wick rotate to exponentially dying modes under the Liouville wall.

[7] This is the first half of the contour obtained in real time thermal field theory in a *mixed* thermal state; see for example section 5 of the second reference in [29] for a recent discussion.

[8] Indeed, the time-dependence of the hamiltonian is only non-adiabatic $1 \sim \frac{\dot\omega}{\omega^2} = \frac{\mu^2 \kappa e^{-\kappa t}}{\omega^3}$ in a small window of time near $t \sim 1/\kappa$. Similar suppression obtains for other measures of nonadiabaticity $\frac{\partial_t^n \omega}{\omega^{n+1}} \xrightarrow{t\to-\infty} (\kappa/\mu)^n e^{-nt/2}$.

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3.4 Singularity structure

So far we focused on two particularly instructive physical quantities: the 1-loop partition function and the genus zero two-point function (Bogoliubov coefficients). Let us now determine the singularity structure of more general amplitudes. This is important in order to complete our assessment of the ability of the tachyon condensate to resolve the spacelike singularity. Namely, if the perturbative amplitudes are finite up to expected divergences associated with physical states (which we will make precise below), then we may conclude that the perturbative string theory is capable of resolving the singularity in the circumstances we have specified (in particular, at weak coupling).

N-point functions at genus zero. As discussed above, the genus zero two-point function describes particle production in the linearized spacetime theory. The singularity structure of general N-point amplitudes can be ascertained from the path integral (3.9) (3.15). In a nontrivial bulk vacuum, such as that derived above in the Euclidean vacuum (3.30), we are interested in a linear combination of vertex operators (3.6) comprised of $\alpha_{\vec{k},n}$ times a negative frequency component plus $\beta_{\vec{k},n}$ times a positive frequency component.

The path integral diverges when a bosonic degree of freedom can go off to infinity unobstructed by the e^{-S_E} factor. As discussed in the introduction, this situation appears in the big bang region of the spacetime in the naive extrapolation of GR, as the space shrinks and the kinetic terms in S_E go away. In our case, where it is positive the tachyon term obstructs this divergence (everywhere in the Heterotic case, and away from the subcritical type 0 regions of the type II system).

There are divergences in the bulk region $X^0 \to \infty$ that are expected in a time dependent S matrix. Generically, the vertex operators provide oscillations suppressing the path integral contribution in this region. However a divergence in $X_0^0 \to +\infty$ appears when the frequencies are such as to cancel this oscillation:

$$\sum \omega_n (\pm 1)_n = 0\,. \tag{3.33}$$

The $\pm$ sign here comes from the presence of both positive and negative frequency modes in the vertex operators.[9] These divergences correspond to expected divergences from physical intermediate states in time dependent systems (see e.g. [42, chapter 9] for a discussion of this).

In [20, 41, 22] subtleties were encountered in a prescription for analytically continuing higher point amplitudes from spacelike Liouville theory. It will be interesting to study these issues in more detail in our case, keeping careful track of the vacuum in which the computations are being done in the various prescriptions for continuing the amplitudes.

Higher loops. Higher loop amplitudes arise from the path integral (3.9) (3.15) defined on a Riemann surface of higher genus h. These contain dependence on the dilaton $\Phi \equiv \Phi_0 + \hat{\Phi}$ where Φ_0 is the constant value in the bulk region. This introduces a factor of $e^{\Phi_0(2h-2)}$

[9]This is another aspect analogous to the situation in Liouville theory (see [17, eq. (87)]).

from the bare bulk string coupling as well as a contribution

$$S_\Phi = \int_\Sigma \hat{\mathcal{R}}^{(2)} \hat{\Phi}[X^0] \tag{3.34}$$

(plus its supersymmetric completion in the Heterotic and Type II cases). Semiclassically $\Phi = \Phi_0$ (i.e. $\hat{\Phi} = 0$) as discussed in (3.7). The dilaton will ultimately get sourced by the tachyon. The corresponding corrections will be generated by the worldsheet path integral, but are suppressed by powers of e^{Φ_0}. Moreover, as in our analysis of the 1-loop vacuum amplitude, the X_0^0 integral reveals that higher genus amplitudes have support limited to the weakly coupled bulk of spacetime.

4. Positively-curved spatial slices

In this section, we generalize our techniques to strings in geometries of the form (1.1) where the Ω are coordinates on higher dimensional spheres. The worldsheet theory will be described by an $O(N)$ model at an energy scale related to X^0 in a way we will specify. In this case there is no topologically-stabilized winding tachyon.[10] The sigma model on spatial slices nevertheless develops a mass gap. We will frame this fact in terms of the discussion of section 1, and investigate the extent to which it can be used to remove the singularity present in the GR approximation. Some aspects of the analysis of section 3 persist. Unlike in the case of flat spatial slices, however, the back-reaction from the velocity of the radion will be harder to control in these examples.

4.1 The mass gap of the $O(N)$ model

Consider N two-dimensional scalar fields arranged into an $O(N)$ vector $\vec{n}$. The partition function of the $O(N)$ model is

$$Z = \int [dn] e^{-\int d^2z R^2 (\partial_\mu \vec{n})^2} \prod_z \delta(n^2(z) - 1)\,. \tag{4.1}$$

A nice way to see the mass term appear is to use a Lagrange multiplier to enforce the delta function localizing the path integral onto a sphere, and large N to simplify the resulting dynamics (see e.g. [43]):

$$Z = \int [dn] \int [d\lambda] e^{-\int d^2z [R^2 \vec{n}(-\partial^2 + i\lambda)\vec{n} + i\lambda]}\,, \tag{4.2}$$

where λ is the Lagrange multiplier field introduced to represent the delta function. Now integrate out n:

$$Z = \int [d\lambda] e^{-N/2\, \mathrm{tr} \ln(-\partial^2 + \lambda) + R^2 \int d^2z \lambda}\,. \tag{4.3}$$

At large N, the λ integral has a well-peaked saddle at

$$\lambda(x) = -im^2\,, \tag{4.4}$$

[10] It might be interesting to consider examples of positively-curved spaces with nonzero π_1 such as S^N/Γ with freely acting Γ.

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where the mass m satisfies

$$R^2 = N \int^{\Lambda} \frac{d^2k}{(2\pi)^2} \frac{1}{k^2+m^2} = \frac{N}{2\pi} \ln \frac{\Lambda}{m} \,. \tag{4.5}$$

Renormalize by defining the running coupling at the scale M by

$$R^2(M) = R_0^2 + \frac{N}{2\pi} \ln \frac{\Lambda}{M} \,. \tag{4.6}$$

Plugging back into the action for n, we have a mass for the n-field which runs like

$$m = M e^{-\frac{2\pi R^2}{N}} \,. \tag{4.7}$$

An alternative UV completion of the model which is sometimes more convenient (and easier to supersymmetrize) gives λ a bare mass: add to the action

$$\delta S = \int a\lambda^2 \,,$$

for a large parameter a. Integrating out λ, this smoothens the delta function, and imposes the $n^2 = R^2$ relation weakly in the UV by a quartic potential.

Supersymmetric $O(N)$ model. Since we wish to study string theories without bulk tachyons, we will need to understand the supersymmetric version of the model. A (1,1) supersymmetric version of the $O(N)$ model has an action

$$S = \int d^2\theta \left(\epsilon_{\alpha\beta} D_\alpha n D_\beta n + \Lambda(n^2 - R^2) + a\Lambda^2 \right);$$

$\alpha, \beta = \pm$, $\Lambda = \lambda + \theta^\alpha \psi_\alpha + \theta^2 F_\lambda$ is now a Lagrange multiplier superfield, and $D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\theta^\beta \sigma^\mu_{\beta\alpha} \partial_\mu$. Note that the type II GSO symmetry acts on Λ as

$$(-1)^{F_L} : \Lambda \mapsto -\Lambda \,, \qquad (-1)^{F_R} : \Lambda \mapsto -\Lambda \,. \tag{4.8}$$

The large N physics is the same as in the bosonic case (see e.g. [44]), exhibiting a mass gap, except now there are two vacua for Λ. When

$$\langle \lambda \rangle = \pm m \tag{4.9}$$

the GSO symmetry is spontaneously broken; the two vacua are identified by the GSO projection. This is just as in the appendix of [3], and it results in a single type zero vacuum. This statement about the GSO projection applies to all N, including the case $N = 2$ of a shrinking circle described in section 2, section 3. In particular, this justifies the comment made in the discussion above (3.21) that the regions of negative potential in the type II worldsheet have a type 0 subcritical GSO projection.

4.2 $\mathbb{CP}^1$ Model

The $(1,1)$ sigma model on S^2 actually has $(2,2)$ supersymmetry. Consider a $(2,2)$ linear sigma model [45] for it. There are two chiral superfields Z^i each with charge one with respect to a single U(1) vectormultiplet. The D-term equation is

$$0 = \sum_{i=1}^{2} |Z^i|^2 - \rho \,. \tag{4.10}$$

Below the scale e of the gauge coupling, this model describes strings propagating on a 2-sphere of radius $R = \sqrt{\rho\alpha'}$. The FI coupling ρ flows logarithmically towards smaller values in the IR:

$$\rho(M) = \rho_0 - 2 \ln \frac{M}{M_0} \,. \tag{4.11}$$

This breaking of scale invariance is in the same $(2,2)$ supermultiplet as an anomaly in the chiral U(1) R-symmetry; only a $\mathbb{Z}_2$ subgroup of this latter group is a symmetry of the quantum theory (this is part of the GSO symmetry in type II theories).

Integrating out the chiral multiplets Z^i leads [45] to an effective twisted superpotential for the vectormultiplet scalar

$$\tilde{W} = 2\Sigma \ln \Sigma - t\Sigma \,. \tag{4.12}$$

Mirror symmetry [46] relates this to a model with one twisted chiral superfield Y, governed by a twisted superpotential

$$\tilde{W} = \Lambda \left(e^{Y} + e^{-Y} \right) , \tag{4.13}$$

where $\Lambda = m e^{-t/2}$, $t = \rho + i\vartheta$. This effective twisted superpotential has isolated massive vacua.

Next let us discuss the GSO projection, to ensure that the relevant operator we are generating is present in the type II theory (as opposed to being a type 0 bulk tachyon). In the type II case, the twisted chiral superpotential must be odd under the chiral GSO actions. This is accomplished by

$$(-1)^{F_L} : Y_2 \mapsto Y_2 + \pi \,, \qquad (-1)^{F_R} : Y_2 \mapsto Y_2 + \pi \,, \tag{4.14}$$

where $Y \equiv Y_1 + iY_2$, or simply $\Sigma \to -\Sigma$.

The twisted superpotential (4.12) has two massive vacua

$$\sigma = \pm e^{-t/2} \,, \tag{4.15}$$

which are permuted by the GSO action. The condensate (4.13) is therefore not a bulk tachyon mode.

We can use this mirror description to further elucidate the physical interpretation of the condensate. It is invariant under the $\mathrm{SU}(2) \simeq \mathrm{SO}(3)$ rotations of the S^2. Since Y_2 is the variable T-dual to the phase of the Zs, from the point of view of the original linear model, (4.13) represents a condensation of winding modes. It is tempting to interpret this as a condensate resembling a ball of rubber-bands wrapping great circles of the small sphere.

A special RG trajectory. In the case $N = 3$ of the two-sphere, where there is a two-cycle in the geometry, there are topologically-charged worldsheet instantons. The contribution to the sum over maps of the sector with winding number n is weighted by $e^{in\theta}$ where $\theta = \int_{S^2} B$ is the period of the NSNS B-field through the two-sphere.

When $\theta = \pi$ this introduces wildly fluctuating signs in the path integral which can result [47] in a critical theory in the IR. In fact, the model flows to the SU(2) WZW model at level one, also known as a free boson at the self-dual radius. *This* model has a topologically-stabilized winding mode, which is exactly massless. At this point, the evolution may be glued onto the analysis of section 2, 3.

4.3 Coupling to string theory

Eternal nothingness is fine if you happen to be dressed for it.
– *Woody Allen*

We need to check whether the mass gap whose origin we have reviewed takes effect *before* large curvature develops. In the example of section 2, 3, the rate of shrinking $\partial_t R$ of the circle was a tunable parameter which we used to control the collapse. In this case, where the spatial curvature exerts a force on R, we will need to reevaluate the behavior of $R(t)$.

In order to do this, we begin at large radius, and use the fact that in this regime, the beta function equations for the worldsheet theory are the same as the gravity equations of motion.

In the case of positive spatial curvature, the Friedmann equation (the hamiltonian constraint) requires a stress-energy source which dominates over the curvature contribution:

$$\left(\frac{\dot{R}}{R}\right)^2 = -\frac{1}{R^2} + G_N \rho \,, \tag{4.16}$$

where ρ is the energy density in non-geometrical sources. The curvature term $-1/R^2$ alone, in the absence of the term from extra sources ρ, would not yield consistent initial data; instead the source term must dominate over the curvature term in the large radius general relativistic regime. This means that unlike the previous two cases of section 2.1, 2.2, we do not classically have a tunable parameter allowing us to slow down the approach to the would-be singularity.

Inclusion of matter. As we mentioned in section 2 3, in the case of positive spatial curvature, the Friedmann equation (4.16) has no real solutions in the absence of matter. We will overcome this problem by including some nonzero radiation energy density on the r.h.s. of (4.16). With $\rho = x/R^N$ (x is a constant and N is the number of spacetime dimensions participating in the FRW space), the maximum radius reached is

$$R_{\max} = l_P x^{\frac{1}{N-2}} \,, \tag{4.17}$$

where $l_P = G_N^{\frac{1}{N-2}}$ is the N-dimensional Planck length. In the curvature-dominated regime,

$$R(t) \sim R_{\max}^{\frac{N-2}{N}} t^{\frac{2}{N}} \,. \tag{4.18}$$

Now we can estimate the time at which the mass gap takes effect. For convenience (as opposed to phenomenology), consider the case $N = 4$, where (4.18) implies $R(t) \sim \sqrt{tR_{\max}}$. Semiclassically, the tachyon term in the worldsheet effective action (4.7) depends on time via the "tachyon" profile

$$\mathcal{T}(t) \sim \mu e^{-R^2/N\alpha'} \,. \tag{4.19}$$

If we assume that the leading effect of the radiation that we added is to fix the evolution of the scale factor (i.e. that it does not couple significantly to the Liouville mode in any other way), we can make a similar estimate to those of the previous sections. In fact, for $N = 4$, the zeromode integral over X^0 is of the same form as (3.23)

$$Z \propto -\ln \mu \,. \tag{4.20}$$

X^0 goes to zero at the would-be bang singularity. The range of X^0 for which the amplitudes have support is $X^0 > \frac{1}{T} \ln \mu/\mu^*$. Increasing μ makes the range of X^0 support of amplitudes smaller. So if we take μ to be large, we can ensure that the lifting of modes occurs in a regime where the kinetic terms have not yet died as we approach the singularity.

Physically, this parameter μ determines the amplitude of the oscillating mode in the bulk and hence its initial behavior in its exponential regime. We are introducing a classical solution with a large amplitude condensate of tachyon even in the initial "bulk" region where the space is larger than string scale, but we expect that these modes will decay once the other states come down.

Particle production. From here the analysis proceeds as in section 3, but the absence of a tunably small rate of growth of the S^3 space leads to a much larger density of produced closed strings. In particular, we again obtain an effective temperature via the periodicity in imaginary time of the condensate. Using the definition

$$\mathcal{T}(t) \propto e^{-Tt}$$

for the effective temperature T, we find (again, for $N = 4$)

$$T \sim \frac{R_{\max}}{\alpha'} \,.$$

Thus, when the cosmology has a phase during which it is bigger than string scale, the effective temperature is larger than the Hagedorn temperature. This is in contrast to the tunably small value of κ we obtained in (3.30) in the case of flat spatial slices.

The upshot is that in this case of positively curved spatial slices, although the mass gap lifts the would-be GR divergences in the worldsheet path integral, a new source of back reaction is generated through copious particle production. It is worth emphasizing that the GR solution alone will lead to particle production of momentum modes, whose back reaction may also correct the background in an important way. We leave this analysis and its potential application to Schwarzchild black hole physics to further work [37].

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5. Discussion

Application to black hole physics. Spacelike singularities appear inside generic black hole solutions of general relativity. The case of a shrinking S^2 described in section 4.1 appears inside the horizon of the Schwarzchild black hole solution in four dimensions (with an additional spatial direction t which is stretching at the same time)

$$ds^2 = -\left(1 - \frac{L_S}{r}\right) dt^2 + \frac{dr^2}{1 - L_S/r} + r^2 d\Omega^2\,, \tag{5.1}$$

where L_S is the Schwarzchild radius. Inside the horizon ($r < L_S$), r is a timelike coordinate.

When the S^2 parameterized by Ω shrinks, the worldsheet path integral develops contributions arising from the mass gap of the corresponding sigma model as discussed in section 4.1. It would be interesting to understand if this might clarify black hole dynamics [37]. These results may also apply to the proposal of [48], where the possibility of postselection on a "nothing" state was explored. The unitarity required in [48] may arise from the unitary evolution along the t direction inside the horizon, generated by the momentum generator inside the horizon.

Other vacua and the shape of the S-matrix. We have focused on a vacuum with no extra excitations above the tachyon background in the initial state. This is motivated by the lifting of closed string degrees of freedom in the presence of the tachyon. However, it would be very interesting to understand if other states are allowed. The analogy with quantum field theory with exponentially growing mass suggests a similar "massing up" of closed string modes in tachyon backgrounds in more general states. Moreover some interesting potential consistency conditions which may restrict the range of allowed states suggest themselves. In particular, a propagator entering the tachyon phase (rather than annihilating against another mode) has the property that semiclassically its worldsheet time ends at a finite value as X^0 goes to infinity. A similar issue arises in formulating the S matrix in the case of pure quantum field theory with an exponentially growing mass (see [38] for a discussion of the issues there); in a generic vacuum there is a sense in which time ends prematurely in this system due to the absence of well defined on shell asymptotic states. Some aspects of the specification of the spacetime quantum state from the worldsheet point of view have been discussed in [35, 49].

In particular, one of the main questions raised by spacelike singularities is that of predictivity. In field theory or GR on a background with a putative big bang singularity, the initial conditions on the fields in the bulk region are ambiguous. If there are other consistent states of the system involving some extra excitations introduced initially and becoming light as the tachyon turns off, then the singularity, while resolved, will not be arbitrarily predictive. It is important to understand the status of all possible states.

Big crunch. Our main computations were done in the vacuum discussed in section 3 with no excitations above the tachyon condensate. In the case of the big bang, this is perhaps a natural choice of initial state. In the case of the big crunch, the methods

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employed in this paper are not yet sufficient to answer the question of what happens starting from an arbitrary initial state. For example, it is interesting to ask what happens if we start with no particles in the bulk. At the level of the genus zero diagrams, we can accomplish this by considering correlation functions of vertex operators which are nontrivial linear combinations of positive and negative frequency modes in the bulk. For the 1-loop and higher genus diagrams, it is an open question here (as in the case of open string tachyons) how the different vacua translate into different prescriptions for the worldsheet path integral.

One aspect of the system is pair production of winding modes themselves as they become massless [50]; this can drain energy from the rolling radius to some extent [51].

Negative spatial curvature. We have discussed the cases of $k = 1$ and $k = 0$. It is natural to ask about the case $k = -1$ where the spatial sections have negative curvature. In this case, at large radius R, the system expands according to the simple relation $R \sim X^0$. Localized tachyon dynamics in some such examples were discussed in [3].

The big bang singularity in the far past in this case is not related by RG flow toward the IR in the matter sector of the corresponding worldsheet sigma model. The direction of flow is opposite; the small radius big bang regime corresponds to the UV. Hence in this case, the big bang resolution may depend on the appropriate UV completion of the sigma model on negatively curved spatial slices.[11]

Cosmology. It will be interesting to see if these methods and results translate into concrete results for more realistic string cosmology. Inflation tends to dilute information about the big bang singularity, but depending on the level of predictivity of the singularity, it may nonetheless play a role. Stretched strings play an important role in our mechanism for resolving the singularity: perhaps there is some relation between them and late-time cosmic strings (whether inside or outside the horizon at late times).

Toward a theory of nothing. *It is the silence between the notes that makes the music;*
it is the space between the bars that holds the tiger.
– *Anonymous*

Our calculations using methods borrowed from Liouville theory exhibit truncation of support of amplitudes to the bulk of spacetime, and hence concretely support the notion of a "nothing" phase in the regime of the tachyon condensate. Conversely, spacetime emerges as the tachyon turns off.

It would be very interesting to characterize this phase and its onset in more detail, for example by unpacking the partition function to analyze its individual contributions. Although perturbative methods exhibit the basic effect, perhaps there is some dual formulation for which the emergence of time as the tachyon turns off is also built in.

[11] One can alternatively add ingredients to metastabilize the system away from this difficult regime [52].

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